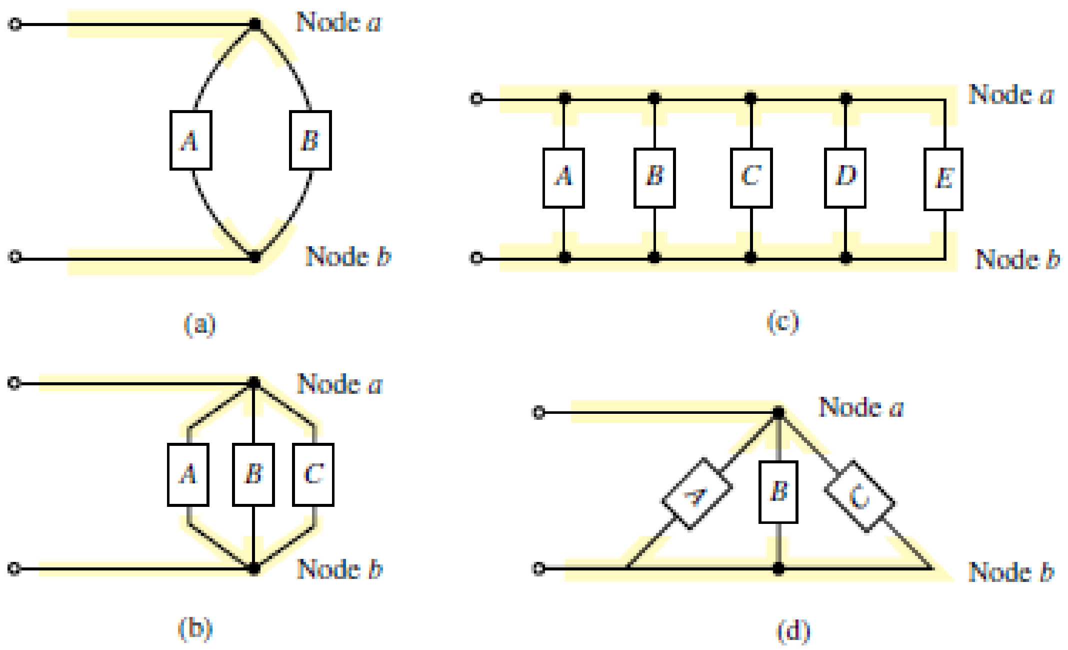
Series and Parallel Circuits

4.1 Parallel circuit

Elements or branches are said to be in a parallel connection when they have exactly two nodes in common as shown in the following Figure.



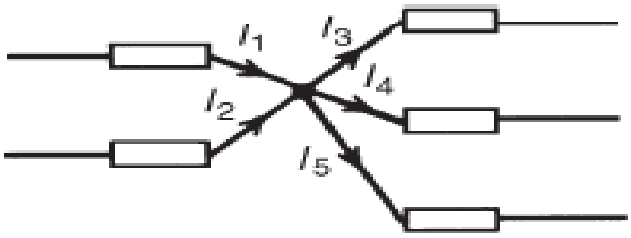
4.2 Kirchhoff's Current Law (KCL)

Kirchhoff's current law (KCL) states the following: The summation of currents entering a node is equal to the summation of currents leaving the node.

$$\sum_{n=1}^{N} i_n = 0$$

where N is the number branches connected to the node and i_n is the n^{th} entering or leaving the node. An alternate way of stating Kirchhoff's current law is as follows: *The sum of currents entering a node is equal to the sum of the current leaving the node.*

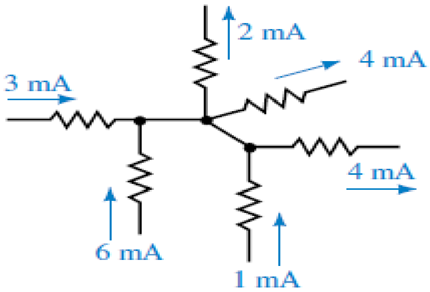
$$\sum I_{entering} = I_{leaving}$$



By applying KCL at the node

$$I_1 + I_2 = I_3 + I_4 + I_5 \qquad or \qquad I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

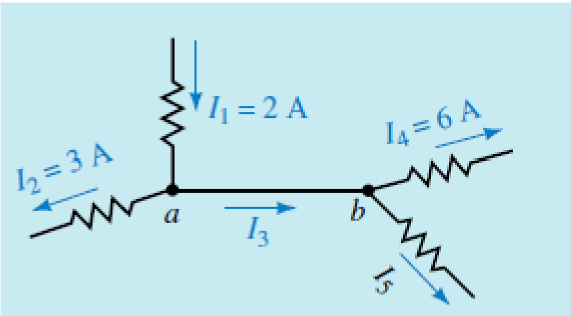
Ex. 1: Verify that KCL applies at the node



$$\sum I_{entering} = I_{leaving}$$

$$3\ mA + 6\ mA + 1\ mA = 2\ mA + 4\ mA + 4\ m$$

Ex. 2: Determine the magnitude and correct direction of the currents I_3 and I_5 for the network



At node a

$$\sum I_{entering} = I_{leaving}$$

$$I_1 = I_2 + I_3$$

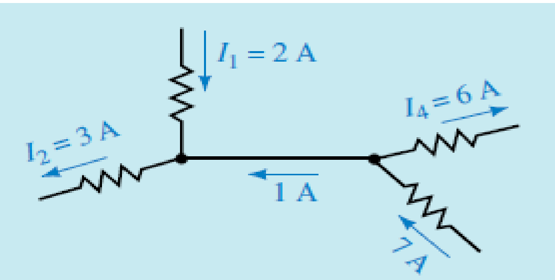
$$2\ A = 3\ A + I_3 \Longrightarrow I_3 = -1\ A = 1\ A \longleftarrow (entering)$$

At node b

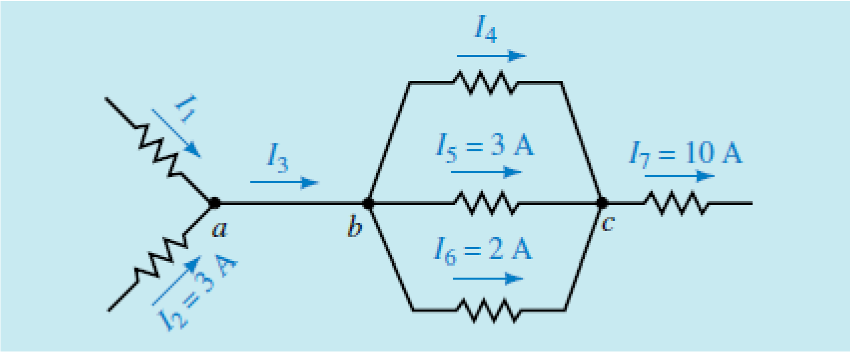
$$\sum I_{entering} = I_{leaving}$$

$$I_3 = I_4 + I_5$$

$$-1\ A = 6\ A + I_5 \Longrightarrow I_3 = -7\ A = 7\ A \longleftarrow (entering)$$



Ex. 3: Determine the magnitudes of the unknown currents for the network



At node a

$$I_1 + I_2 = I_3 \Longrightarrow I_1 + 3\,A = I_3$$

At node b

$$I_3 = I_4 + I_5 + I_6 \Longrightarrow I_3 = I_4 + 3\,A + 2\,A$$

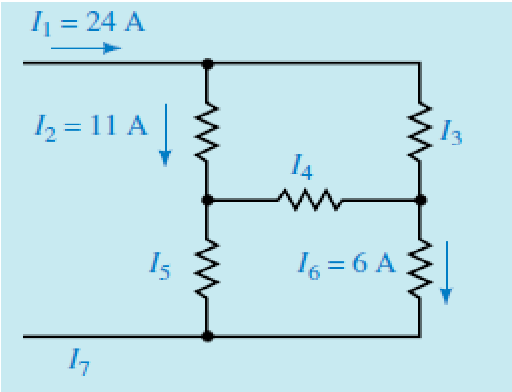
At node c

$$I_4 + I_5 + I_6 = I_7 \Longrightarrow I_4 + 3\,A + 2\,A = 10\,A \Longrightarrow I_4 = 5A$$

$$I_3 = I_4 + 3\,A + 2\,A = 10\,A$$

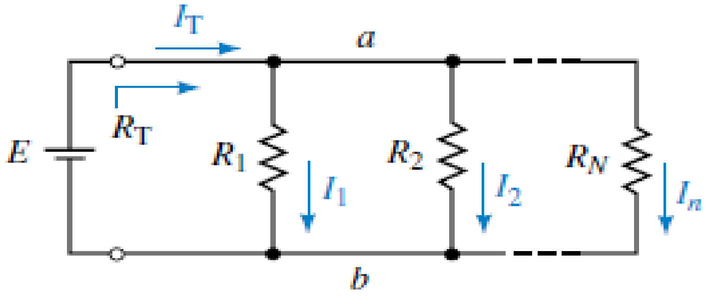
$$I_1 + 3\,A = I_3 \Longrightarrow I_1 = 10\,A - 3\,A = 7\,A$$

H.W 1: Determine the unknown currents in the network (*Ans.* $I_3 = 13\,A \downarrow I_4 = 7\,A \longleftarrow I_5 = 18\,A \downarrow I_7 = 24\,A \leftarrow$)



4.3 Resistors in Parallel

A simple parallel circuit is constructed by combining a voltage source with several resistors



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \quad \text{(siemens, S)}$$

Since conductance was defined as the reciprocal of resistance,

$$G_T = G_1 + G_2 + \ldots\ldots + G_N \qquad \text{(s)}$$

If there are n equal resistors in parallel, each resistor, R, has the same conductance, G then the total conductance is

$$G_T = n\,G$$

And the total resistance is

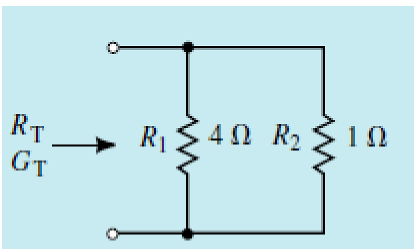
$$R_T = \frac{R}{n} \qquad \Omega$$

An important effect of combining parallel resistors is that the resultant resistance will always be smaller than the smallest resistor in the combination.

The total resistance for two resistors (R_1 and R_2) in parallel is

$$R_T = \frac{R_1 R_2}{R_1 + R_2} \quad \Omega$$

Ex. 4: Solve for the total conductance and total equivalent resistance of the circuit



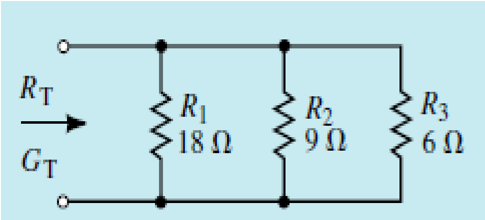
$$G_T = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$G_T = \frac{1}{4} + \frac{1}{1} = 1.25\ S$$

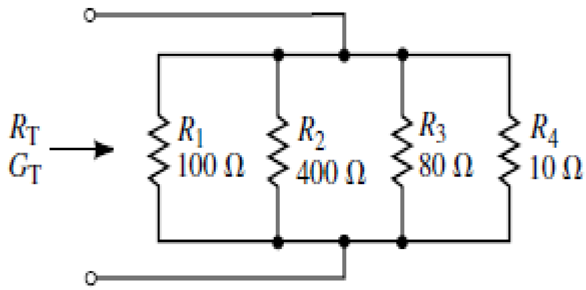
$$G_T = \frac{1}{R_T} \Longrightarrow R_T = 0.8\ \Omega$$

Note that: $R_T < R_1 < R_2$

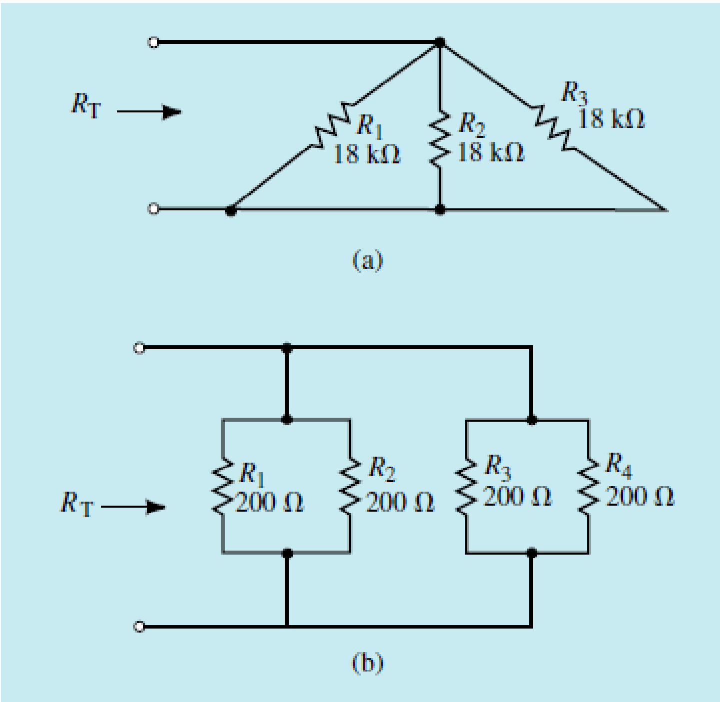
H.W 2: Determine the conductance and resistance of the network (Ans. G_T= 0.333 S and R_T= 3 Ω)



H.W 3: Determine the conductance and resistance of the network (Ans. G_T = 0.125 S and R_T = 8 Ω)

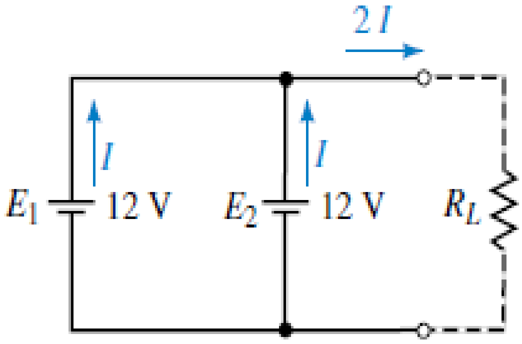


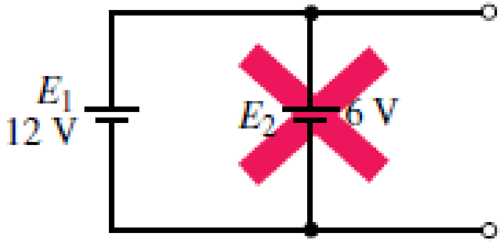
H.W 4: Determine the resistance of the networks (Ans. (a) R_T = 6 Ω (b) R_T = 50 Ω)



4.4 Voltage Sources in Parallel

When two equal potential sources are connected in parallel, each source will deliver half the required circuit current. Note that voltage sources of different potentials should never be connected in parallel.





4.5 Current Divider Rule (CDR)

The current divider rule (CDR) is used to determine how current entering a node is split between the various parallel resistors connected to the node. Consider the network of parallel resistors

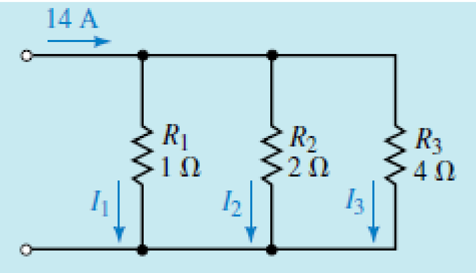
$$I_X = \frac{R_T}{R_X} I_T$$

If the network consists of only two parallel resistors, then the current through each resistor

$$I_1 = \frac{R_2}{R_2+R_1} I_T$$

$$I_2 = \frac{R_1}{R_2+R_1} I_T$$

Ex. 5: For the network, determine the currents I_1, I_2, and I_3



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{1}{1} + \frac{1}{2} + \frac{1}{4} = 1.75$$

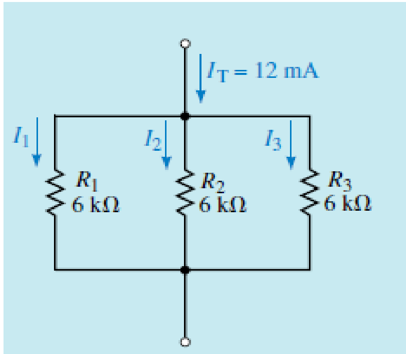
$$R_T = \frac{1}{1.75} = 0.571\ \Omega$$

$$I_1 = \frac{R_T}{R_1} I_T \Rightarrow I_1 = \frac{0.571}{1}(14A) = 8\ A$$

$$I_2 = \frac{R_T}{R_2} I_T \Rightarrow I_2 = \frac{0.571}{2}(14A) = 4\ A$$

$$I_3 = \frac{R_T}{R_3} I_T \Rightarrow I_3 = \frac{0.571}{4}(14A) = 2\ A$$

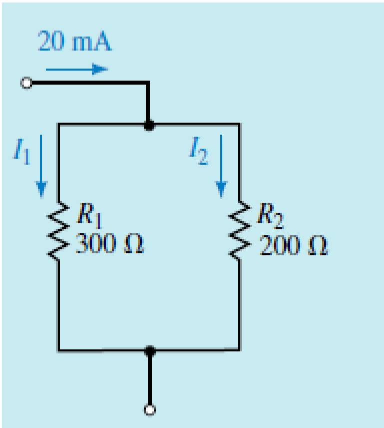
Ex. 6: For the network, determine the currents I_1, I_2, and I_3



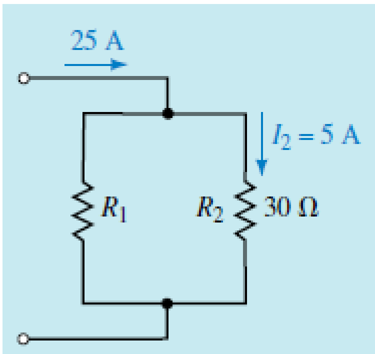
Since $R_1 = R_2 = R_3$ then

$$I_1 = I_2 = I_3 = \frac{I_T}{3} = 4\ mA$$

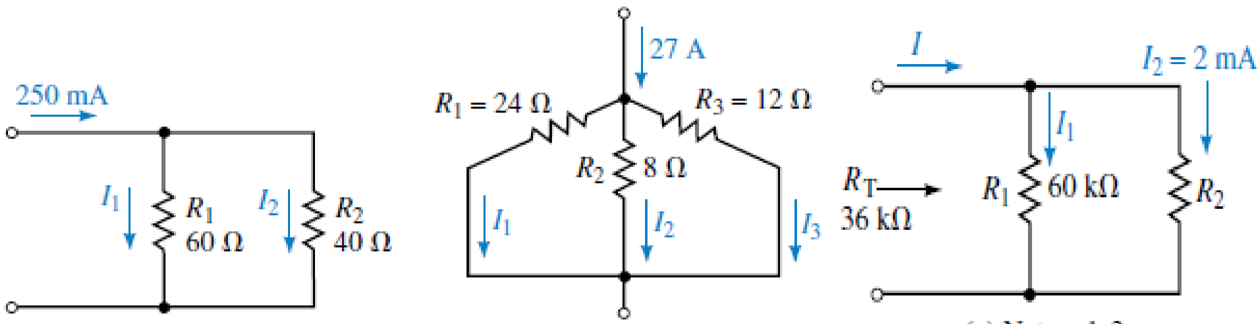
H.W 5: For the network, determine the currents I_1 and I_2 (Ans. $I_1 = 8$ mA $I_2 = 12$ mA)



H.W 6: For the network, determine the currents R_1 (Ans. $R_1 = 7.5\ \Omega$)

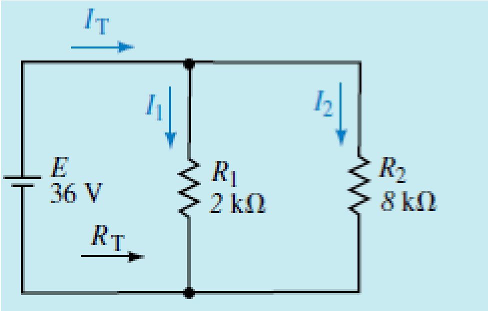


H.W 7: Calculate the unknown currents for the networks (Ans. (a) $I_1 = 100$ mA $I_2 = 150$ mA (b) $I_1 = 4.5$ A $I_2 = 13.5$ A $I_3 = 9$ A (c) $I_1 = 3$ mA $I = 5$ mA)



(a) Network 1 (b) Network 2 (c) Network 3

4.6 Analysis of Parallel Circuits



Ex. 7: For the circuit, determine the followings:

a. R_T

b. I_T

c. I_1 and I_2

d. Power delivered by the voltage source

e. Power dissipated by the resistors

a.

$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(2\ k\Omega)(8\ k\Omega)}{2\ k\Omega + 8\ k\Omega} = 1.6\ k\Omega$$

b.

$$I_T = \frac{E}{R_T} = \frac{36\ V}{1.6\ k\Omega} = 22.5\ mA$$

c.

$$I_1 = \frac{R_T}{R_1} I_T = \frac{1.6\ k\Omega}{2\ k\Omega}(\ 22.5\ mA) = 18\ mA\ \ \boldsymbol{Or}\ \ I_1 = \frac{E}{R_1} = \frac{36\ V}{2\ k\Omega} = 18\ mA$$

$$I_2 = \frac{R_T}{R_2} I_T = \frac{1.6\ k\Omega}{8\ k\Omega}(\ 22.5\ mA) = 4.5\ mA\ \ \boldsymbol{Or}\ \ I_2 = \frac{E}{R_2} = \frac{36\ V}{8\ k\Omega} = 4.5\ mA$$

$$\boldsymbol{Or}\ I_2 = I_T - I_1 = 22.5\ mA - 18\ mA = 4.5\ mA$$

d.

$$P_{Source} = EI_T = (36\ V)(22.5\ mA) = 810\ mW$$

e.

$$P_1 = {I_1}^2 R_1 = (18\ mA)^2(2\ k\Omega) = 648\ mW$$

$$P_2 = {I_2}^2 R_2 = (4.5\ mA)^2(8\ k\Omega) = 162\ mW$$

$$P_T = P_{Source} = P_1 + P_2 = 648\ mW + 162\ mW = 810\ mW$$

H.W 8: For the following circuit determine:

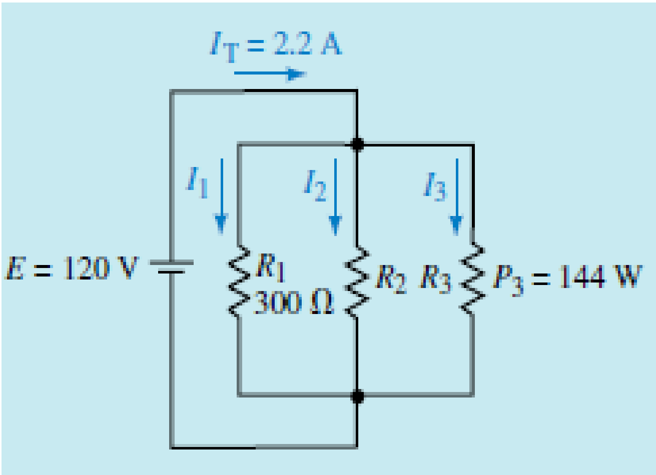
a. Find the currents I_1, I_2, and I_3. (Ans. $I_1 = 0.4$ A $I_2 = 0.6$ A $I_3 = 1.2$ A)

b. Determine the values of the unknown resistors R_2 and R_3 (Ans. $R_2 = 200\ \Omega$ $R_3 = 100\ \Omega$)

c. Calculate the total power delivered by the voltage source. (Ans. $P_T = 264$ W)

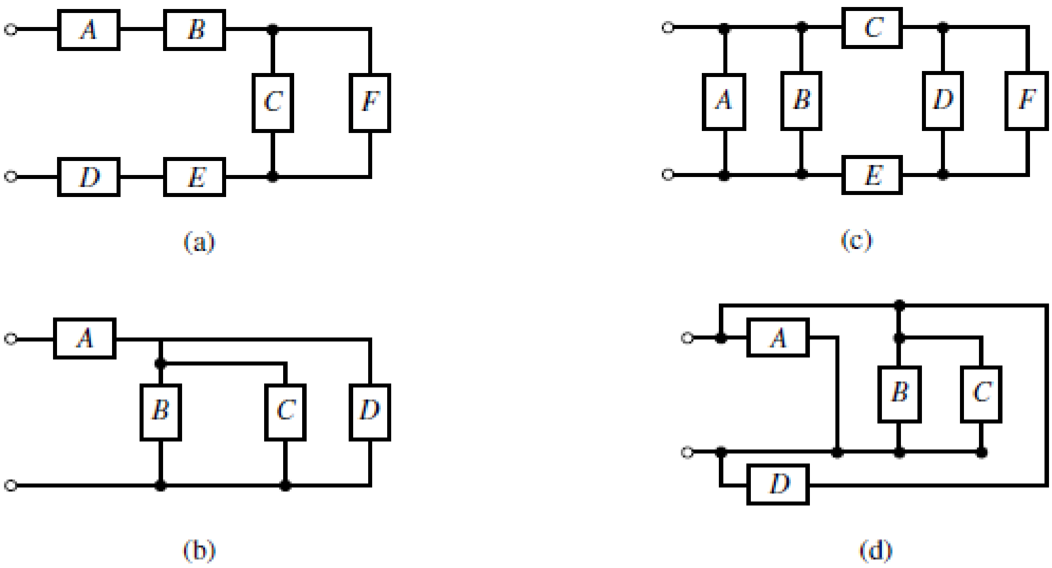
d. Calculate the power dissipated by each resistor. (Ans. $P_1 = 48$ W $P_2 = 72$ W $P_3 = 144$ W)

e. Verify that the power dissipated is equal to the power delivered by the voltage source.



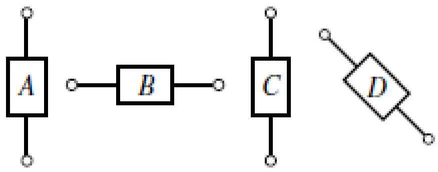
Exercises:

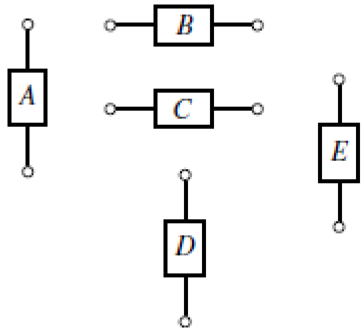
1. Indicate which of the elements are connected in parallel and which elements are connected in series.



Ans.
a. A and B are in series; D and E are in series; C and F are parallel
b. B, C, and D are parallel
c. A and B are parallel; D and F are parallel
d. A, B, C, and D are parallel

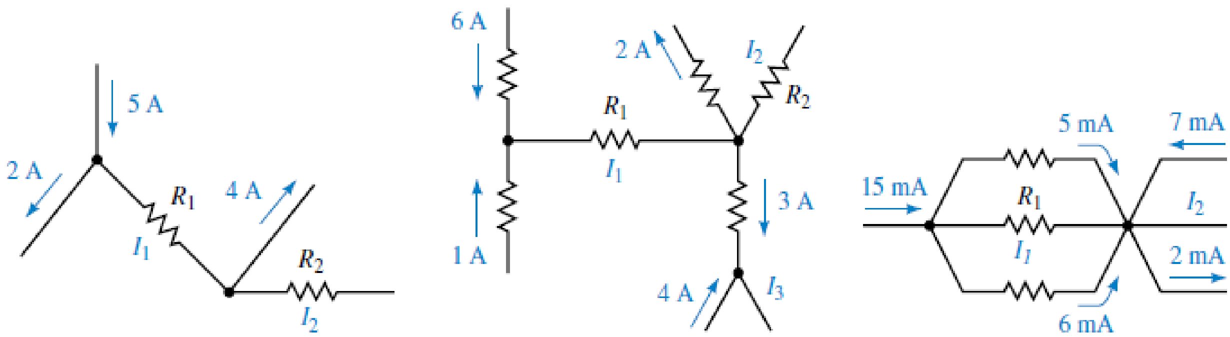
2. Without changing the component positions, show the way of connecting all the elements in parallel.





3. Determine the magnitudes and directions of the indicated currents in each of the networks

Ans. (a) $I_1 = 3$ A $I_2 = -1$ A (b) $I_1 = 7$ A $I_2 = 2$ A $I_3 = -7$ A (c) $I_1 = 4$ mA $I_2 = 20$ mA

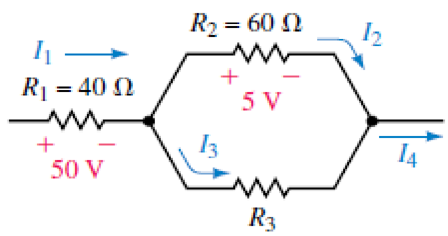


(a) (b) (c)

4. Consider the network:

a. Calculate the currents I_1, I_2, I_3, and I_4. (Ans. $I_1 = 1.25$ A $I_2 = 0.0833$ A $I_3 = 1.167$ A $I_4 = 1.25$ A)

b. Determine the value of the resistance R_3. (Ans. $R_3 = 4.29\ \Omega$)

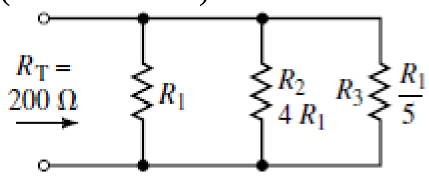


5. For the network find:

a. Calculate the values of R_1, R_2, and R_3 so that the total resistance of the network is 200 Ω. (Ans. $R_1 = 1250\ \Omega$ $R_2 = 5$ kΩ $R_3 = 250\ \Omega$)

b. If R_3 has a current of 2 A, determine the current through each of the other resistors. (Ans. $IR_1 = 0.4$ A $I_{R2} = 0.1$ A)

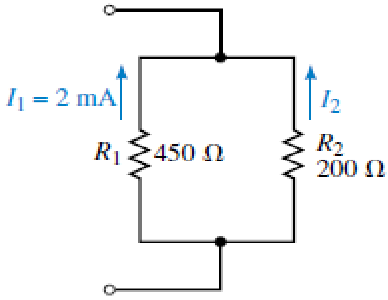
c. How much current must be applied to the entire network? (Ans. $I = 2.5$ A)



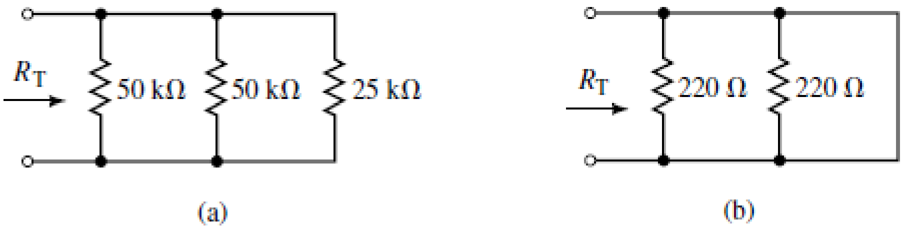
6. For the network:

a. Find the voltages across R_1 and R_2. (Ans. $V_1 = V_2 = 900$ mV)

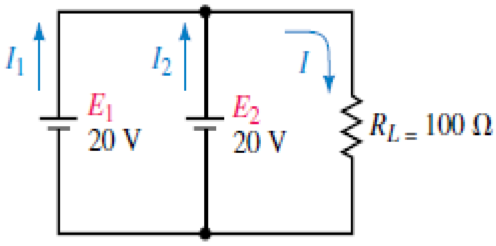
b. Determine the current I_2. (Ans. $I_2 = 4.5$ A)



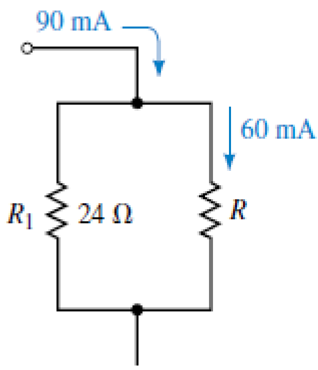
7. Without using a pencil, paper, or a calculator, determine the total resistance of each network. (Ans. (a) $R_T = 12.5$ kΩ $R_T = 0$ Ω)



8. Two 20-V batteries are connected in parallel to provide current to a 100 Ω load as shown in the following Figure. Determine the current in the load and the current in each battery. (Ans. I=0.2 A $I_1 = I_2 = 0.1$ A)



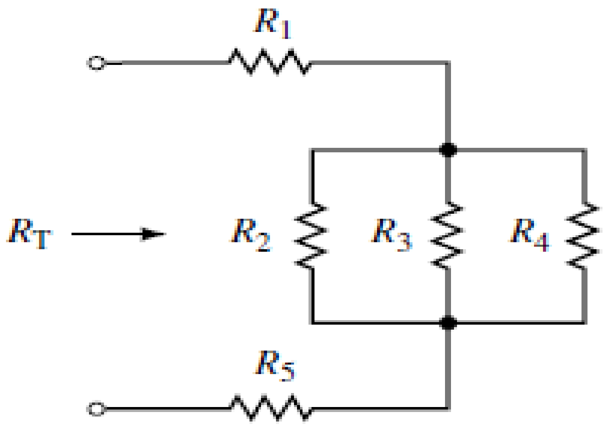
9. Determine the unknown resistance in the network. (Ans. R= 12 Ω)



4.7 The Series-Parallel Network

Most circuits are neither simple series circuits nor simple parallel circuits, but rather a combination of the two. Kirchhoff's voltage and current laws are applied to the analysis of series-parallel networks.

In order to analyze a complicated circuit, it is important to be able to recognize which elements are in series and which elements or branches are in parallel. Consider the network of resistors

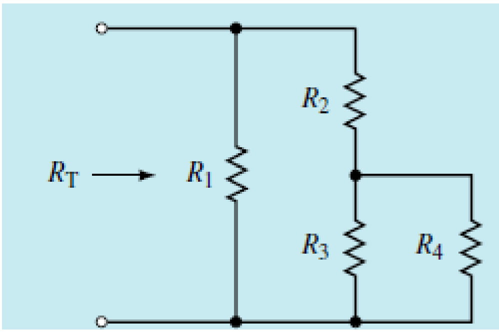


The resistors R_2, R_3, and R_4 are in parallel. This parallel combination is in series with the resistors R_1 and R_5. The total resistance may now be written as follows:

$R_T = R_1 + (R_2 \,//\, R_3 \,//\, R_4) + R_5$

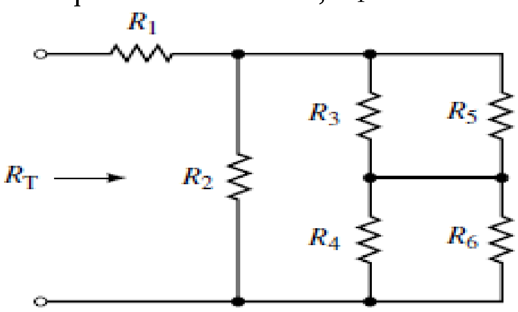
Ex. 8: For the network, write an expression for the total equivalent resistance, R_T

$R_T = R_1 \,//\, (R_2 + (R_3 \,//\, R_4))$



H.W 9: For the network, write an expression for the total equivalent resistance, R_T

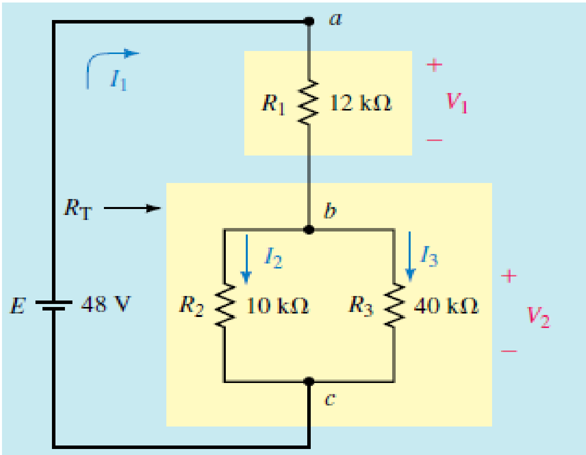
Ans. $R_T = R_1 + (R_2 \,//\, ((R_4 \,//\, R_6) + (R_3 \,//\, R_5)))$



4.8 Analysis of Series-Parallel Circuits

The analysis of series-parallel dc networks with a single source usually required that the resistance "seen" by the source (R_T) be determined by combining series and parallel elements until a single equivalent resistance remains. The source current can then calculated and the remaining currents of the network determined by working back through the network.

Ex. 9: Consider the circuit



a. Find R_T.

$R_T = R_1 + (R_2 \,//\, R_3)$

$R_T = 20\ k\Omega$

b. Calculate I_1, I_2, and I_3

$$I_1 = \frac{E}{R_T} = \frac{48\ V}{20\ k\Omega} = 2.4\ mA$$

$$I_2 = I_1 \frac{R_3}{R_2 + R_3} = \frac{40\ k\Omega}{10\ k\Omega + 40\ k\Omega}(2.4\ mA) = 1.92\ mA$$

$$I_3 = I_1 \frac{R_2}{R_2 + R_3} = \frac{10\ k\Omega}{10\ k\Omega + 40\ k\Omega}(2.4\ mA) = 0.48\ mA$$

c. Determine the voltages V_1 and V_2.

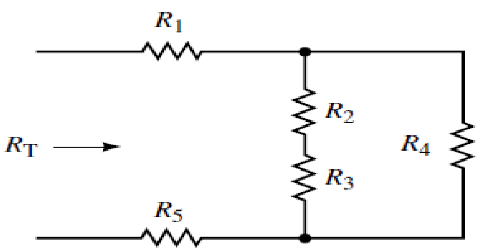
$$V_1 = I_1 R_1 = (2.4\ mA)(12\ k\Omega) = 28.8\ V$$

$$V_2 = I_2 R_2 = I_3 R_3 = (1.92\ mA)(10\ k\Omega) = 19.2\ V$$

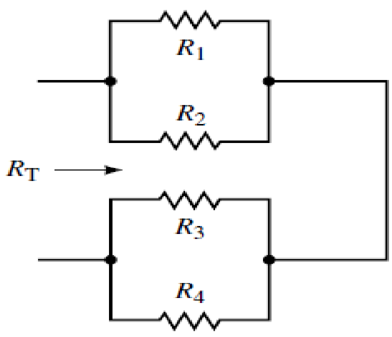
Exercises

1. For the networks, determine which resistors and branches are in series and which are in parallel. Write an expression for the total resistance, R_T. Ans. (a) $R_T = R_1 + ((R_2 + R_3) \,//\, R_4) + R_5$

(b) $R_T = (R_1 \,//\, R_2) + (R_3 \,//\, R_4)$



(a)



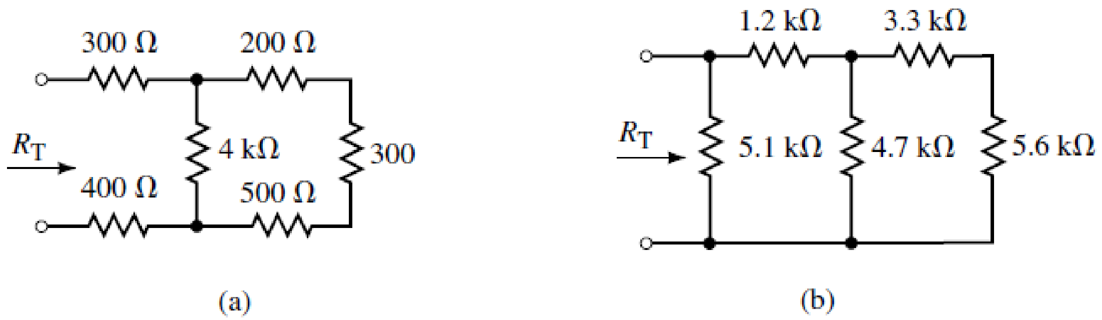
(b)

2. Resistor networks have total resistances as given below. Sketch a circuit which corresponds to each expression.

(a) $R_T = (R_1 // R_2 // R_3) + (R_4 // R_5)$

(b) $R_T = R_1 + (R_2 // R_3) + (R_4 // (R_5 + R_6))$

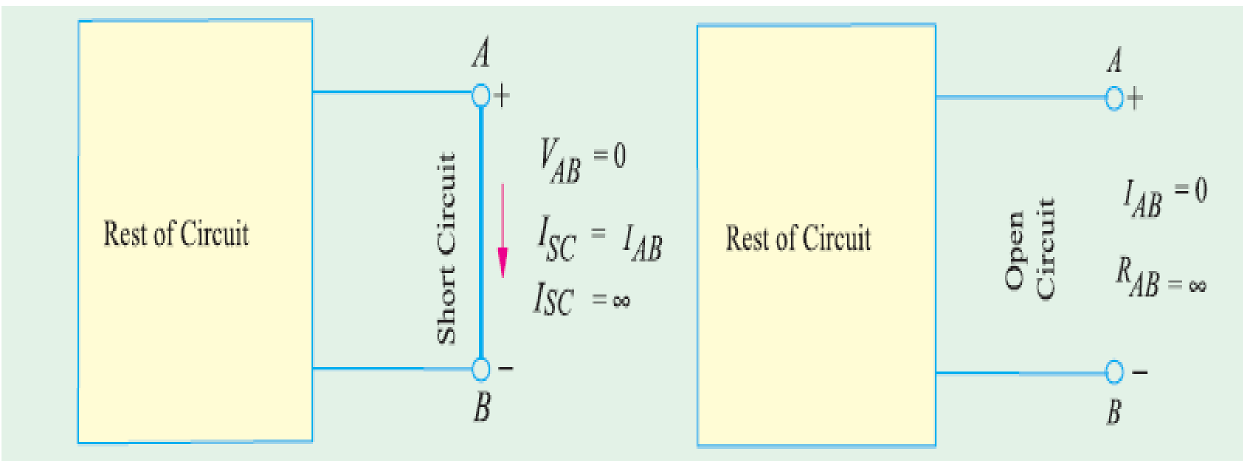
3. Determine the total resistance of each network (Ans. (a) 1500 Ω (b) 2.33 $k\Omega$)



4.9 Short and Open Circuits

When two points of circuit are connected together by a thick metallic wire, they are said to be *short-circuited.* Since 'short' has practically zero resistance, it gives rise to two important facts:

- No voltage can exist across it because $V = IR = I \times 0 = 0$
- Current through it (called short-circuit current) is very large (theoretically, infinity)

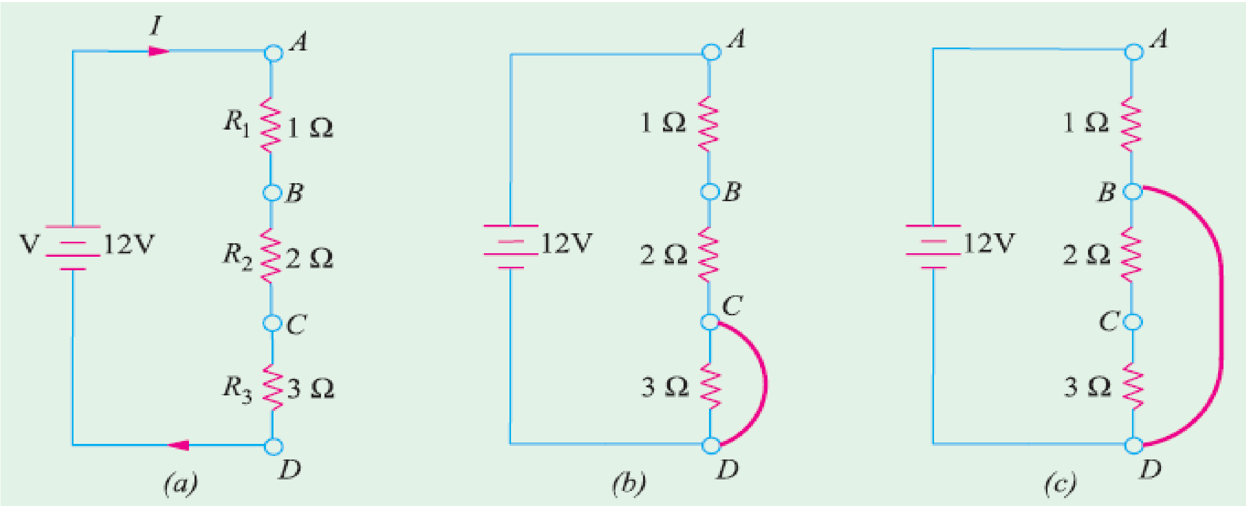


Two points are said to be open-circuited when there is no direct connection between them. Obviously, an 'open' represents a break in the continuity of the circuit. Due to this break

- Resistance between the two points is infinite.
- There is no flow of current between the two points.

4.9.1 Shorts in a Series Circuit

Since a dead (or solid) short has almost zero resistance, it causes the problem of excessive current which, in turn, causes power dissipation to increase many times and circuit components to burn out.



In Figure (*a*) is shown a normal series circuit where

$V = 12$ V, $R = R_1 + R_2 + R_3 = 6\ \Omega$

$I = V/R = 12/6 = 2$ A, $P = I^2R = (4)(6) = 24$ W

In Figure (*b*), 3 Ω resistor has been shorted out by a resistanceless copper wire so that $R_{CD} = 0$.

Now,

$R = 1 + 2 + 0 = 3\ \Omega$, $I = 12/3 = 4$ A and $P = (16)(3) = 48$ W.

Figure (*c*) shows the situation where both 2 Ω and 3 Ω resistors have been shorted out of the circuit.

In this case,

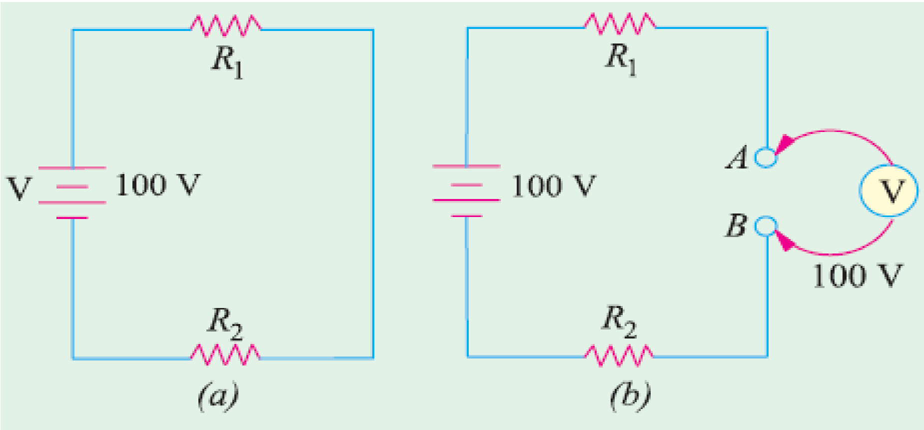
$R = 1\ \Omega$, $I = 12/1 = 12$ A and $P = (122)(1) = 144$ W

Because of this excessive current (6 times the normal value), connecting wires and other circuit components can become hot enough to ignite and burn out.

4.9.2 Opens in a Series Circuit

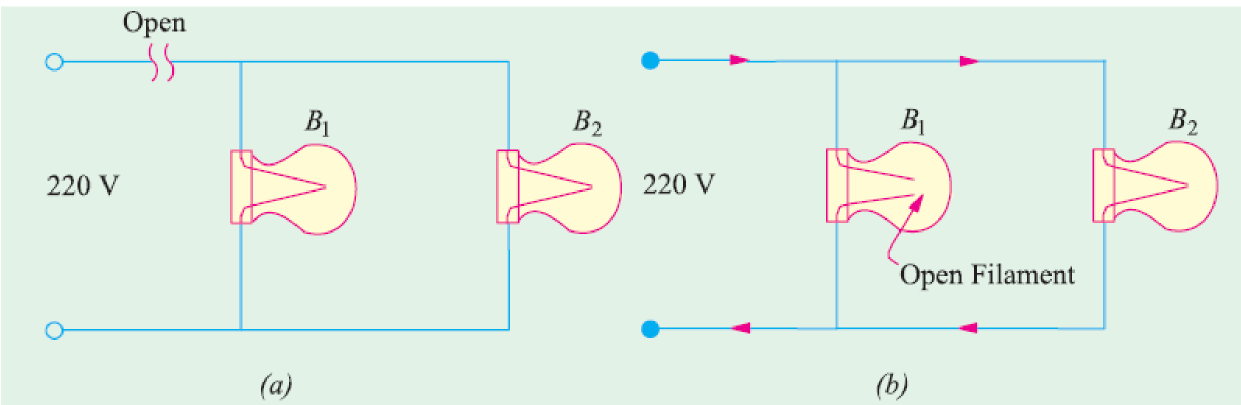
In a normal series circuit like the one shown in the following Figure (*a*), there exists a current flow and the voltage drops across different resistors are proportional to their resistances. If the circuit becomes 'open' anywhere, following two effects are produced :

- Since 'open' offers infinite resistance, circuit current becomes zero. Consequently, there is no voltage drop across R_1 and R_2.
- Whole of the applied voltage is felt across the 'open' i.e. across terminals A and B (Figure b)



4.9.3 Opens in a Parallel Circuit

Since an 'open' offers infinite resistance, there would be no current in that part of the circuit where it occurs. In a parallel circuit, an 'open' can occur either in the main line or in any parallel branch.
As shown in the following Figure (a), an open in the main line prevents flow of current to all branches. Hence, neither of the two bulbs glows. However, full applied voltage is available across the open.



In Figure (*b*), 'open' has occurred in branch circuits of B_1. Since there is no current in this branch, B_1 will not glow. However, as the other bulb remains connected across the voltage supply, it would keep operating normality. It may be noted that if a voltmeter is connected across the open bulb, it will read full supply voltage of 220 V.

4.9.4 'Shorts' in Parallel Circuits

Suppose a 'short' is placed across R_3 in the following Figure. It becomes directly connected across the battery and draws almost infinite current because not only its own resistance but that of the connecting wires *AC* and *BD* is negligible. Due to this excessive current, the wires may get hot enough to burn out unless the circuit is protected by a fuse.

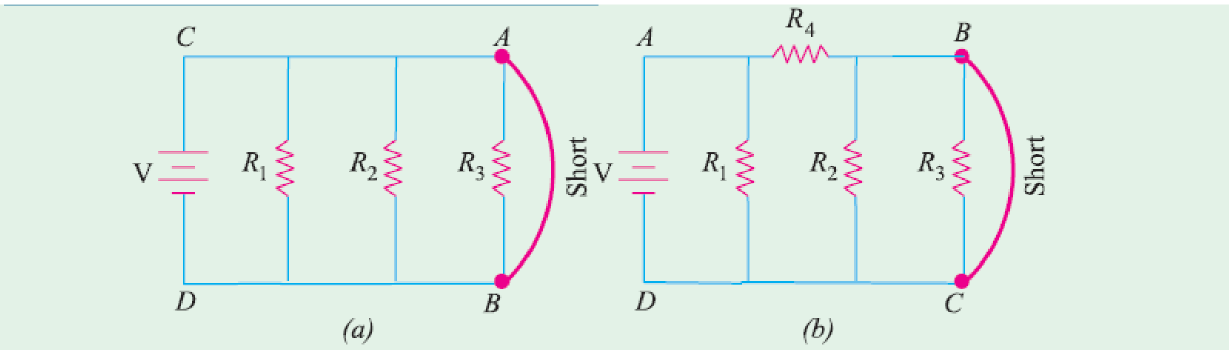


Fig. 1.43

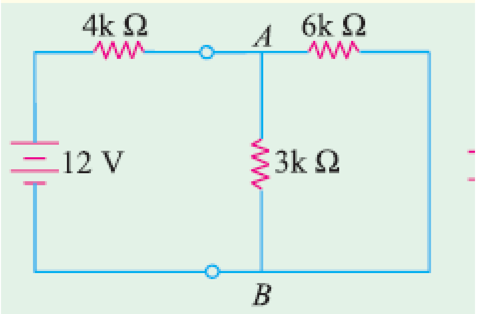
Following points about the circuit of Figure (*a*) are worth noting.

- Not only is R_3 short-circuited but both R_1 and R_2 are also shorted out i.e. short across one branch means short across all branches.
- There is no current in shorted resistors. If there were three bulbs, they will not glow.
- The shorted components are not damaged, the three bulbs would glow again when circuit is restored to normal conditions by removing the short-circuited.

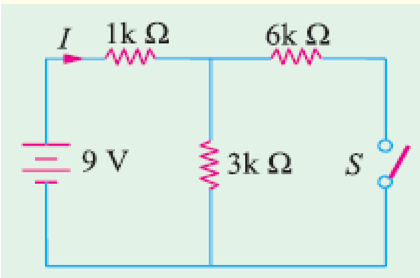
From Figure (b) that a short-circuit across R_3 may short out R_2 but not R_1 since it is protected by R_4.

Exercises:

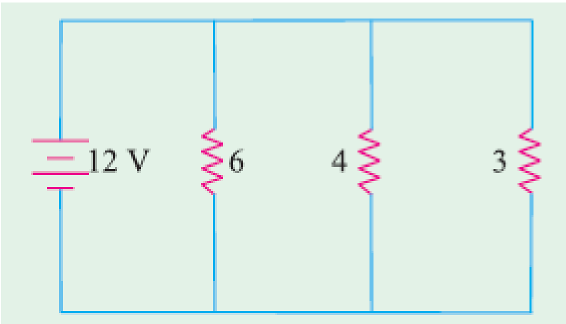
1. Find the current and power supplied by the battery to the circuit of the following Figure when a 'short' occurs across terminals *A* and *B*. [Ans. : 3 mA; 36 mW]



2. Compute the values of battery current *I* and voltage drop across 6 kΩ resistor of the following Figure when switch *S* is open. [Ans. : 2.25 mA; 0V]



3. A fault has occurred in the circuit of the following Figure. One resistor has burnt out and has become an open. Which is the resistor if current supplied by the battery is 6 A ? All resistances are in ohm. [Ans: 4 Ω]



4. In the following Figure if resistance between terminals *A* and *B* measures 1000 Ω, which resistor is opencircuited. All conductance values are in milli-siemens (mS). [Ans: 0.8 mS]