Series Circuits

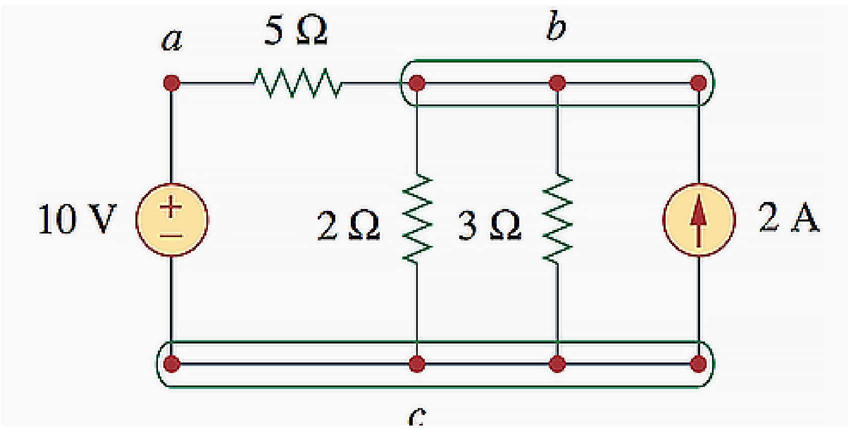
3.1 Branches, Nodes and Loops

A **branch** represents a single element such as a voltage source or a resistor. In other words, a branch represents any two-terminal element.

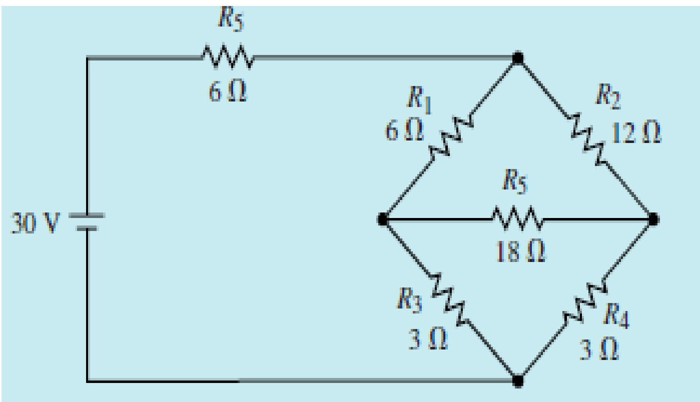
A **node** is the point of connection between two or more branches. If a connecting wire connects two nodes, the two nodes constitute a single node. The following circuit Five branches and three nodes a, b, and c.

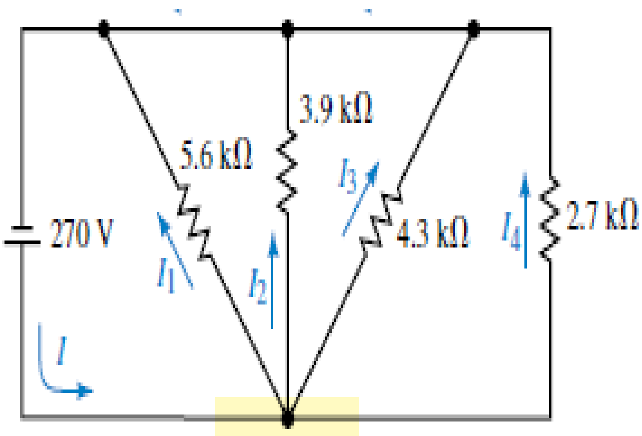
A **Loop** is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.

The following circuit has five branches, three nodes a, b, and c and six loops



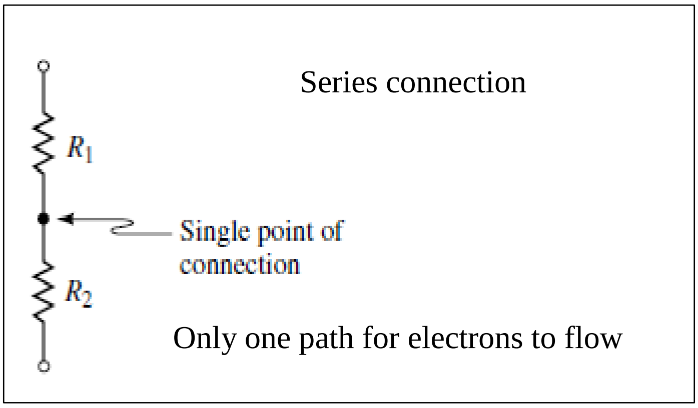
H.W1: Determine the number of branches and nodes in the following circuits



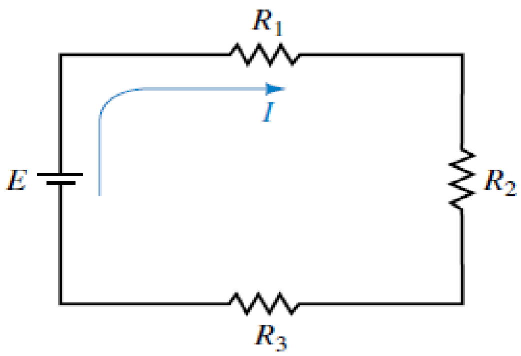


3.2 Series Circuits

An **electric circuit** is the combination of any number of sources and loads connected in any manner which allows charge to flow. The basic idea of a series connection is that components are connected at a single point to form a single path for electrons to flow as shown in the following Figure.



A **series circuit** is constructed by combining various elements in series, as shown in the following Figure. Current will leave the positive terminal of the voltage source, move through the resistors, and return to the negative terminal of the source.



3.3 Kirchhoff's Voltage Law (KVL)

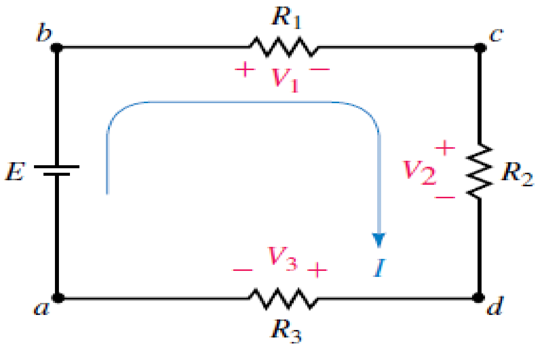
Kirchhoff's voltage law (KVL) is one of the most important laws of electricity which states the following: *The summation of voltage rises and voltage drops around a closed loop is equal to zero.* This may be stated as follows:

$$\sum_{m}^{M} v_m = 0 \; for \; aclosed \; loop$$

where M is the number of voltages in the loop and v_m is the m^{th} voltage.

An alternate way of stating Kirchhoff's voltage law is as follows: *The summation of voltage rises is equal to the summation of voltage drops around a closed loop.*

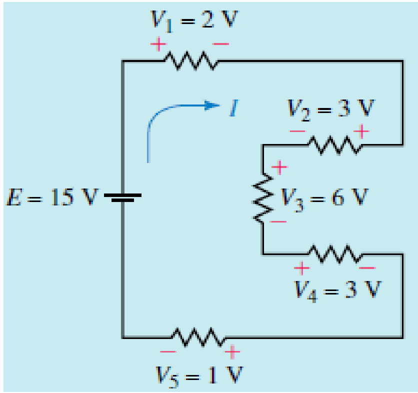
$$\sum E_{rises} = \sum v_{drops} \; for \; a \; closed \; loop$$



By applying Kirchhoff's voltage law around the closed loop (abcda) shown in the Figure above,

$$E - V_1 - V_2 - V_3 = 0 \quad \text{Or} \quad E = V_1 + V_2 + V_3$$

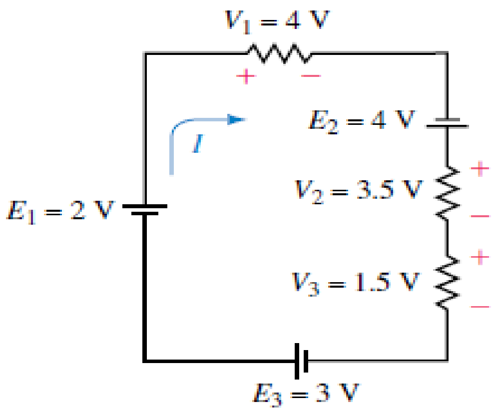
Ex.1: Verify Kirchhoff's voltage law for the circuit



$$E - V_1 - V_2 - V_3 - V_4 + -V_5 = 0$$

$$15 - 2 - 3 - 6 - 3 + 1 = 0$$

Ex. 2: Verify Kirchhoff's voltage law for the circuit



$$E_1 - V_1 + E_2 - V_2 - V_3 + E_3 = 0$$

$$2 - 4 + 4 - 3.5 - 1.5 + 3 = 0$$

3.4 Resistors in Series

For the following circuit

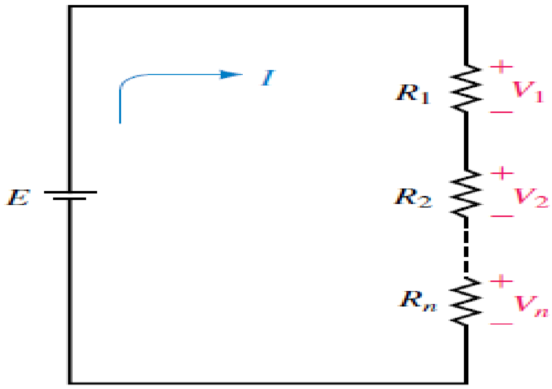
$$I = I_{R1} = I_{R2} = \ldots\ldots = I_{Rn}$$

By KVL $\quad E = V_1 + V_2 + \ldots\ldots + V_n$

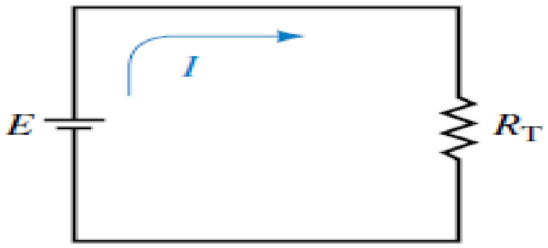
$$E = I\,R_1 + I\,R_2 + \ldots\ldots + I\,R_n$$

$$E = I\,(R_1 + R_2 + \ldots\ldots + R_n)$$

$$E = I\,R_T$$



By replacing all the resistors with an equivalent total resistance, R_T



So the total resistance of the *n* series resistors is

$$R_T = R_1 + R_2 + R_3 + \ldots\ldots + R_n$$

$$R_T = \frac{E}{I}$$

If each of the *n* resistors has the same value, then the total resistance is determined as

$$R_T = n\,R$$

The power dissipated by each resistor is determined as

$$P_1 = V_1 I = \frac{V_1^2}{R_1} = I_1^2 R_1 \quad [watts, W]$$

$$P_2 = V_2 I = \frac{V_2^2}{R_2} = I_2^2 R_2 \quad [watts, W]$$

$$P_n = V_n I = \frac{V_n^2}{R_n} = I_n^2 R_n \quad [watts, W]$$

The power delivered by the source is given as

$$P_T = EI \quad [watts, W]$$

The delivered power by the source is equal to the total power dissipated by all the resistors

$$P_T = P_1 + P_2 + \cdots + P_n$$

Ex. 3: For the following circuit, find:

a. Total resistance, RT.

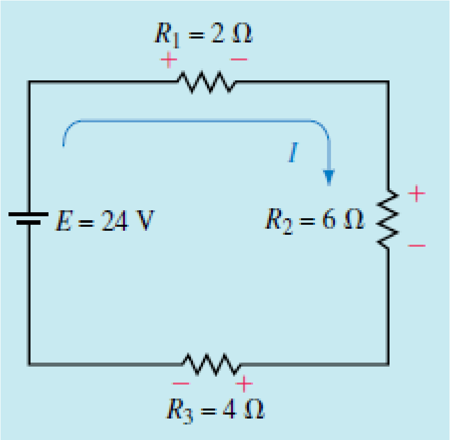
b. Circuit current, I.

c. Voltage across each resistor.

d. Power dissipated by each resistor.

e. Power delivered to the circuit by the voltage source.

f. Verify that the power dissipated by the resistors is equal to the power delivered to the circuit by the voltage source.



a. $R_T = 2\Omega + 6\Omega + 4\Omega = 12\Omega$

b. $I = \frac{24V}{12\Omega} = 2\text{A}$

c. $V_1 = (2\text{A})(2\Omega) = 4\text{V}$

$V_2 = (2\text{A})(6\Omega) = 12\text{V}$

$V_3 = (2\text{A})(4\Omega) = 8\text{V}$

d. $P_1 = (2A)^2(2\Omega) = 8\text{W}$

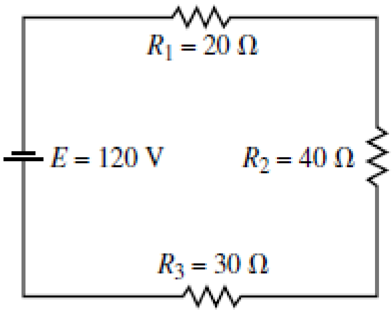
$P_2 = (2A)^2(6\Omega) = 24\text{W}$

$P_3 = (2A)^2(4\Omega) = 16\text{W}$

e. $P_T = (24V)(2\text{A}) = 48\text{W}$

f. $P_T = 8\ W + 24\ W + 16\ W = 48\text{W}$

H.W 2: For the following circuit, find



a. Total resistance, RT.

b. The direction and magnitude of the current, I.

c. Voltage across each resistor and its polarity.

d. Power dissipated by each resistor.

e. Power delivered to the circuit by the voltage source.

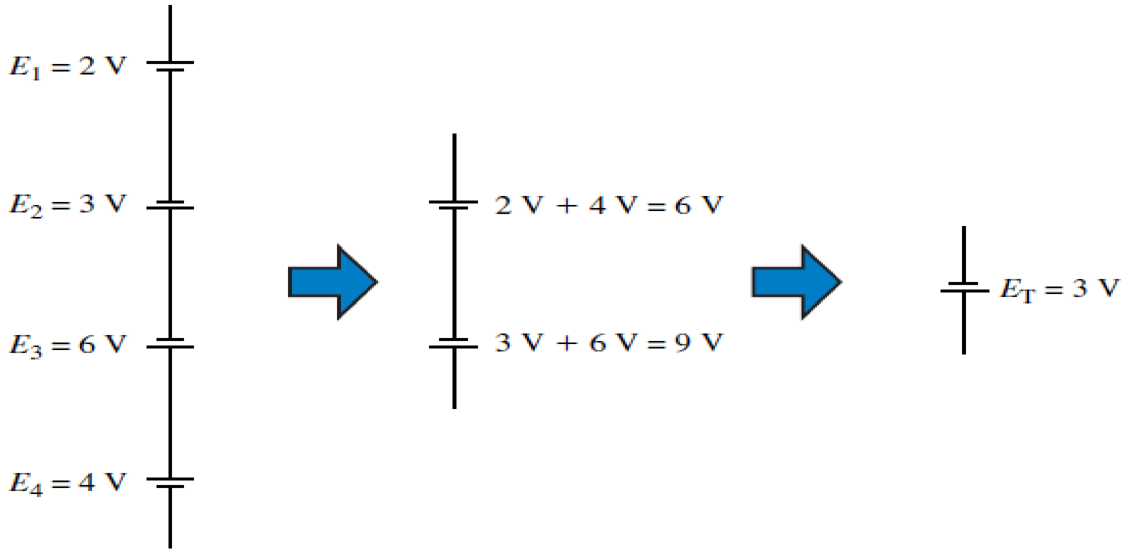
f. Show that the power dissipated is equal to the power delivered.

(Ans. **a.** 90Ω **b.** 1.33 A counterclockwise **c.** V_1 =26.7 V, V_2 =53.33 V, V_3 =40 V **d.** P_1 =35.6 W , P_2 =71.1 W, P_3 =53.3W **e.** P_T =160 W **f.** $P_1 + P_2 + P_3$=160W

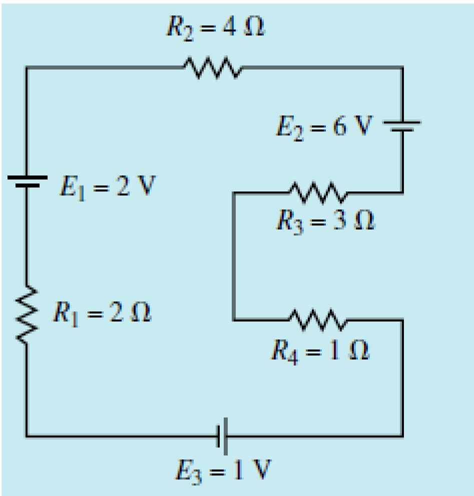
H.W 3: Three resistors, R_1, R_2, and R_3, are in series. Determine the value of each resistor if R_T =42 kΩ, $R_2 = 3R_1$, and $R_3 = 2R_2$ (Ans. R_1=4.2kΩ, R_2=12.6 kΩ, R_3=25.2 kΩ)

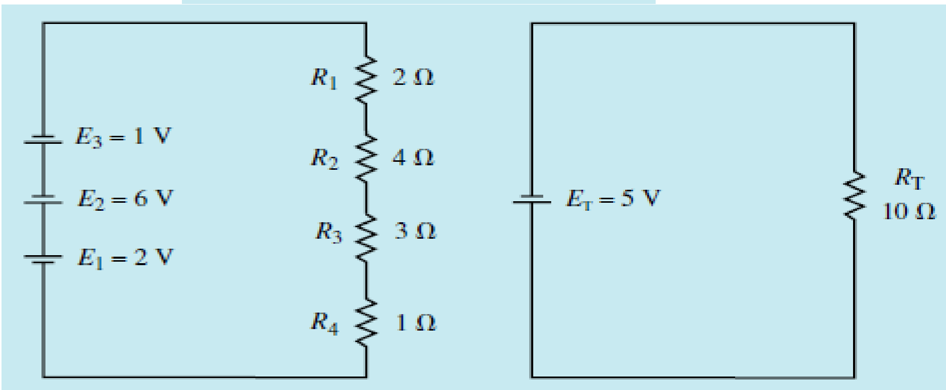
3.4 Voltage Sources in Series

Voltage source can be connected in a series as shown in the following Figure to increase or decrease the total voltage applied to a system. The net voltage is determined simplify by summing the source with same polarity and subtracting the total of the source with the opposite polarity. The net polarity is the polarity of the larger sum.



Ex. 4: Determine the direction and magnitude of the current in the following circuit





$$I = \frac{E_T}{R_T} = \frac{6\,V + 1\,V - 2\,V}{2\,\Omega + 4\,\Omega + 3\,\Omega + 1\,\Omega} = \frac{5\,V}{10\,\Omega} = 0.5\,A \downarrow$$

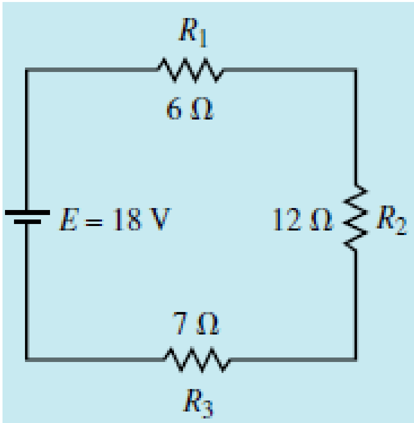
3.5 The Voltage Divider Rule (VDR)

The voltage across a single element or a combination of the elements in a series circuit can be determined without first finding the current if the voltage divider rule is applied.

$$V_x = \frac{E\,R_x}{R_t}$$

where V_x is the voltage across R_x, E is the impressed voltage the series elements and R_t is the total resistance of the series circuit.

Ex. 5: Using VDR to determine the voltage across each of resistor in the following circuit



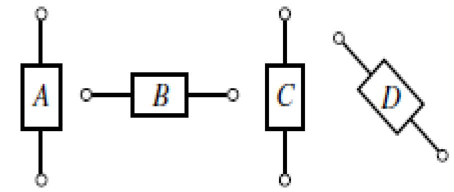
$$V_1 = \frac{(18\,V)(6\,\Omega)}{(6\,\Omega + 12\,\Omega + 7\,\Omega)} = 4.32\,V$$

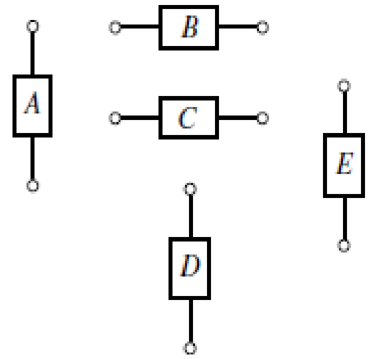
$$V_2 = \frac{(18\,V)(12\,\Omega)}{(6\,\Omega + 12\,\Omega + 7\,\Omega)} = 8.64\,V$$

$$V_3 = \frac{(18\,V)(7\,\Omega)}{(6\,\Omega + 12\,\Omega + 7\,\Omega)} = 5.04\,V$$

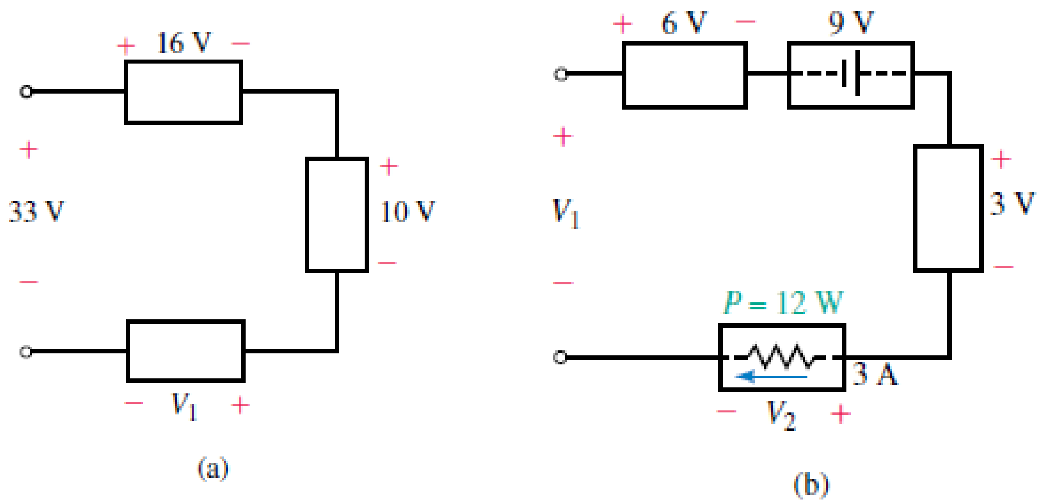
Exercises:

1. Without changing the component positions, show the way of connecting all the elements in series.

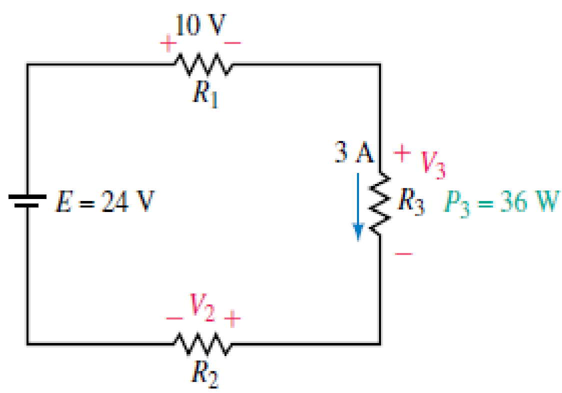




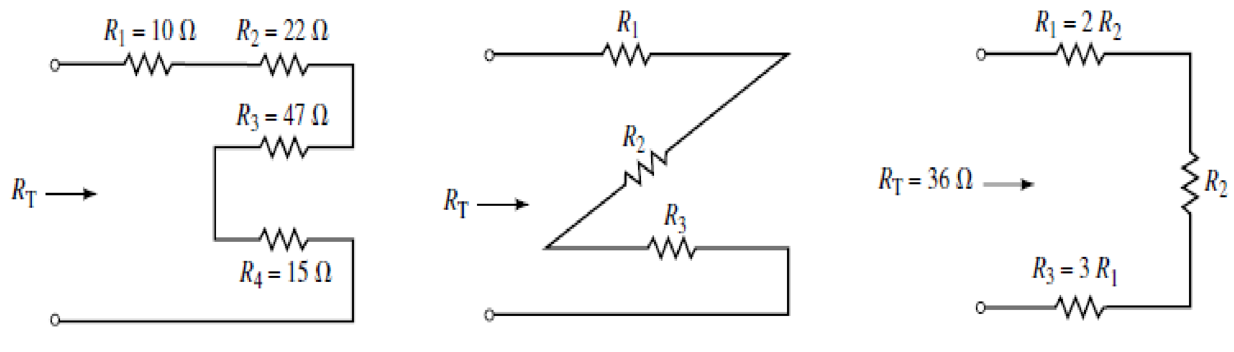
2. Determine the unknown voltages in the networks (Ans. (a) V_1= 7 V (b) V_2= 4 V and V_1=4 V)



3. Determine the unknown voltages in the circuit (Ans. (a) V_3= 12 V (b) V_2= 2 V)

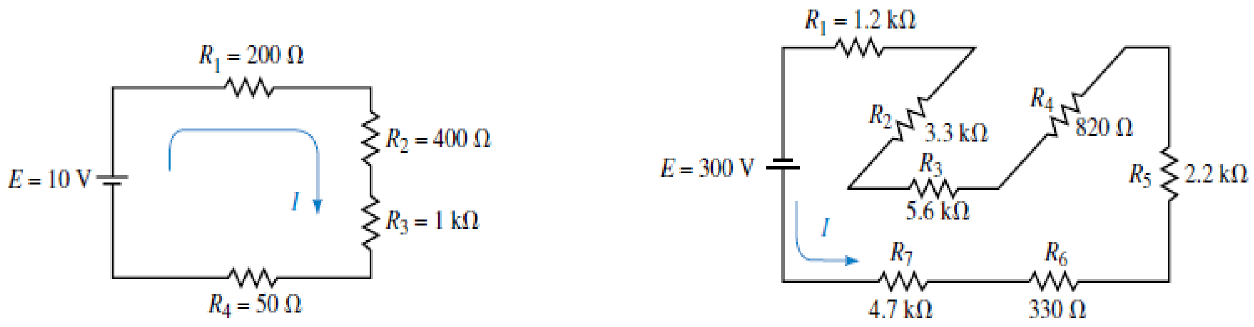


4. Determine the unknown resistance in each of the networks (Ans. (a) $R_T = 94\Omega$ (b) $R_T = 15\Omega$ (c) $R_1 = 8\Omega$ $R_2 = 4\ \Omega$ $R_3 = 24\Omega$)



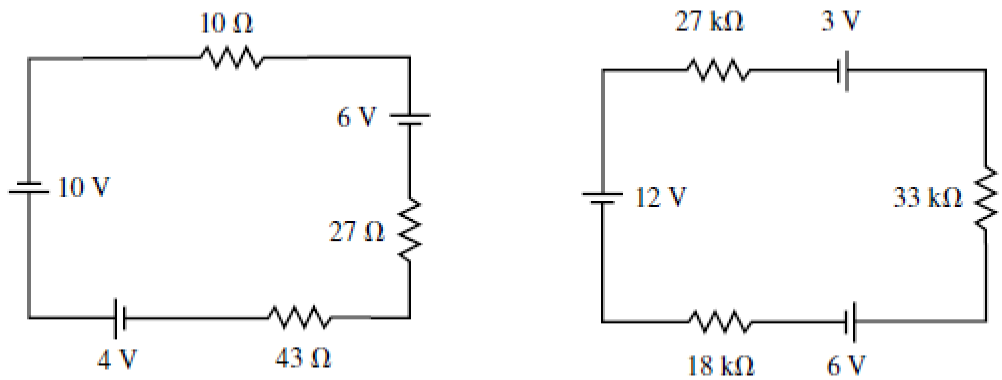
(a) (b) Each resister is 5Ω (c)

5. For the circuits, determine the total resistance, R_T, and the current, I. (Ans. (a) $R_T = 1650\ \Omega$ I=6.06 mA (b) $R_T = 18.15\ \Omega$ I=16.5 mA)



(a) Circuit 1 (b) Circuit 2

6. Determine the value and direction of the circuit in each circuit (Ans. (a) I = 0.15 A (b) I=0.115 mA)



(a) Circuit 1 (b) Circuit 2