Chapter Four
Magnetic circuits

Introduction

It may be defined as the complete closed path which is followed by magnetic flux. The law of magnetic circuit are quite similar to (but not the same as) those of the electric circuit. Consider a solenoid or a (toroid) iron ring having a magnetic path of *l* metre and a coil of N turns carrying I amperes on it as in Fig.4.1, the dotted lines represent the flux set up within a ring.

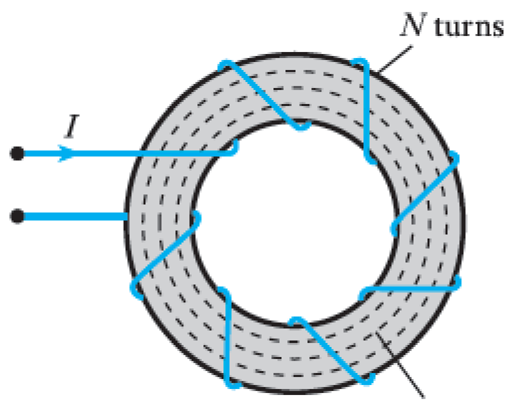


Fig.4.1. A toroid

Magnetomotive force and magnetic field strength

In an electric circuit, the current is due to the existence of an force. By analogy, we may say that in a magnetic circuit the magnetic flux is due to the existence of a magnetomotive force (m.m.f.) caused by a current flowing through one or more turns. The value of the m.m.f. is proportional to the current and to the number of turns, and is descriptively expressed in ampere-turns.

Magnetomotive force

Symbol: F Unit: ampere turns (At): If a current of I amperes flows through a coil of N turns, as shown in Fig. 4.1, the magnetomotive force F is the total current linked with the magnetic circuit, namely IN amperes. The magnetomotive force per unit length of the magnetic circuit is termed the magnetic field strength and is represented by the symbol H. Thus, if the length of the magnetic circuit of Fig. 4.1 is *l* metres, then

$$H = IN/l \quad \text{..........(4.1)}$$

Magnetic field strength

Symbol: H Unit: ampere-turns per metre (At/m).

and $H = \frac{F}{l}$

where $F = NI$ ……….(4.2)

Magnetic flux

The lines of magnetic field is called magnetic flux. It's symbol is φ and it's unit is (wb).

The flux Density (B)

It is given by the flux passing per unit area through a plane at right angles to the flux. It is usually designated by the capital letter B and is measured in weber/meter^2.

It ΦWb is the total magnetic flux passing normally through an area of A m^2, then

$$B = \Phi/A \text{ Wb/m}^2 \text{ or tesla (T)}$$

Absolute and Relative Permeabilities of a Medium

The ratio B/H in a material is always constant and is equal to (μ) which is a measure of material conductivity for magnetic flux. Every medium is supposed to possess two permeabilities :

(*i*) absolute permeability (μ) and (*ii*) relative permeability (μ_r).

The relative permeability of vacuum with reference to itself is unity.

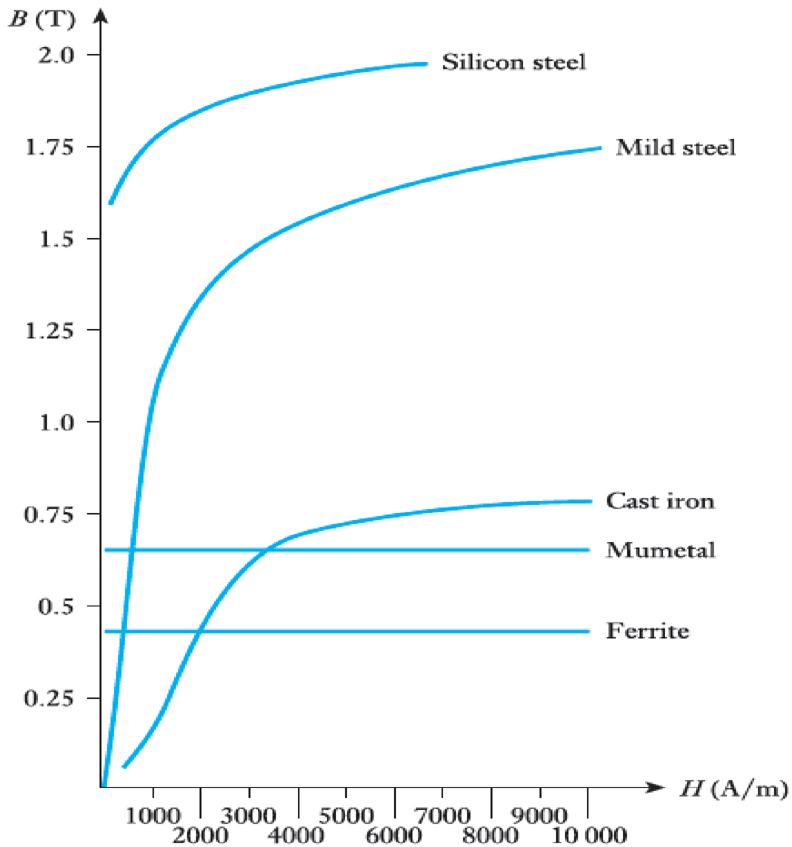
Hence, for free space,

absolute permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m

relative permeability $\mu_r = 1$.

Now, take any medium other than vacuum. If its relative permeability, as compared to vacuum is μ_r, then its absolute permeability is $\mu = \mu_0 \mu_r$ H/m.

Figure below show the B-H curve for different materials.



$$B = \mu_r \mu_0 H \quad \text{..........(4.3)}$$

Example 4.1: A coil of 200 turns is wound uniformly over a wooden ring having a mean circumference of 600 mm and a uniform cross-sectional area of 500 mm^2. If the current through the coil is 4.0 A, calculate

(a) the magnetic field strength;

(b) the flux density;

(c) the total flux.

Solution:

(a) Mean circumference = 600 mm = 0.6 m.

$$\therefore \quad H = 4 \times 200/0.6 = 1333 \text{ A/m}$$

(b) Flux density $= \mu_0 H = 4\pi \times 10^{-7} \times 1333$

$$= 0.001\,675 \text{ T} = 1675\ \mu\text{T}$$

(c) Cross-sectional area $= 500 \text{ mm}^2 = 500 \times 10^{-6} \text{ m}^2$

$$\therefore \quad \text{Total flux} = 1675\ [\mu\text{T}] \times (500 \times 10^{-6})\ [\text{m}^2] \equiv \mathbf{0.838\ \mu Wb}$$

Reluctance

Let us consider a ferromagnetic ring having a cross-sectional area of A square metres and a mean circumference of l metres (Fig.4.1), wound with N turns carrying a current I amperes, then total flux (Φ) = flux density × area

$$\therefore \quad \Phi = BA \qquad [4.4]$$

and m.m.f. (F) = magnetic field strength × length.

$$\therefore \quad F = Hl \qquad [4.5]$$

Dividing equation [4.4] by [4.5], we have

$$\frac{\Phi}{F} = \frac{BA}{Hl} = \mu_r \mu_0 \times \frac{A}{l}$$

$$\therefore \quad \Phi = \frac{F}{\frac{l}{\mu_0 \mu_r A}}$$

where

$$\frac{F}{\Phi} = \frac{l}{\mu_0 \mu_r A} = S \qquad [4.6]$$

S is the reluctance of the magnetic circuit where

$$F = \Phi S \qquad [4.7]$$

and

$$S = \frac{l}{\mu_0 \mu_r A} \qquad [4.8]$$

Ohm's law for a magnetic circuit

m.m.f. = flux × reluctance

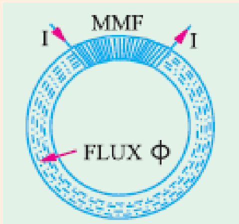
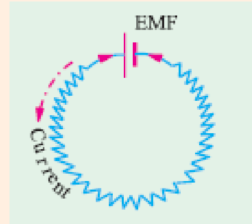
F = ΦS

or NI = ΦS

It is clear that m.m.f. (F) is analogous to e.m.f. (E) and flux (Φ) is analogous to current (I) in a d.c. resistive circuit, where

e.m.f. = current × resistance

E = IR.

Magnetic Circuit	Electric Circuit
 Fig.4.3.	 Fig.4.4.
1. Flux = $\frac{\text{m.m.f.}}{\text{reluctance}}$	Current = $\frac{\text{e.m.f.}}{\text{resistance}}$
2. M.M.F. (ampere-turns)	E.M.F. (volts)
3. Flux Φ (webers)	Current I (amperes)
4. Flux density B (Wb/m^2)	Current density (A/m^2)
5. Reluctance $S = \frac{l}{\mu A}\left(= \frac{l}{\mu_0 \mu_r A}\right)$	resistance $R = \rho \frac{l}{A} = \frac{l}{\rho A}$
6. Permeance (= 1/reluctance)	Conductance (= 1/resistance)
7. Reluctivity	Resistivity
8. Permeability (= 1/reluctivity)	Conductivity (= 1/resistivity)
9. Total m.m.f. $= \Phi S_1 + \Phi S_2 + \Phi S_3 + ...$	9. Total e.m.f. $= IR_1 + IR_2 + IR_3 + ...$

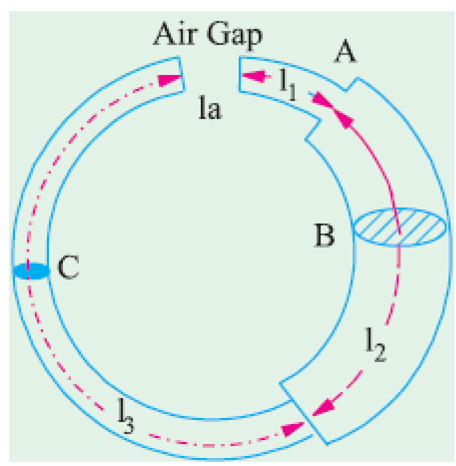
Composite Series Magnetic Circuit

Fig.4.5 is shown a composite series magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths and one air gap (μr = 1). Each path will have its own reluctance. The total reluctance is the sum of individual reluctances as they are joined in series.

$$\therefore \quad \text{total reluctance} = \sum \frac{l}{\mu_0 \mu_r A}$$

$$= \frac{l_1}{\mu_0 \mu_{r_1} A_1} + \frac{l_2}{\mu_0 \mu_{r_2} A_2} + \frac{l_3}{\mu_0 \mu_{r_3} A_3} + \frac{l_a}{\mu_0 A_g}$$

$$\therefore \quad \text{flux } \Phi = \frac{\text{m.m.f.}}{\dfrac{l}{\mu_0 \mu_r A}}$$



Example 4.2: A magnetic circuit comprises three parts in series, each of different cross-sectional area (c.s.a.). They are:

(a) a length of 80 mm and c.s.a. 50 mm^2,

(b) a length of 60 mm and c.s.a. 90 mm^2,

(c) an air gap of length 0.5 mm and c.s.a. 150 mm^2.
A coil of 4000 turns is wound on part (b), and the flux density in the air gap is 0.30 T. Assuming that all the flux passes through the given circuit, and that the relative permeability μ_r is 1300, estimate the coil current to produce such a flux density.

Solution:

$$\Phi = B_c A_c = 0.3 \times 1.5 \times 10^{-4} = 0.45 \times 10^{-4}\,\text{Wb}$$

$$F_a = \Phi S_a = \Phi \cdot \frac{l_a}{\mu_0 \mu_r A_a}$$

$$= \frac{0.45 \times 10^{-4} \times 80 \times 10^{-3}}{4\pi \times 10^{-7} \times 1300 \times 50 \times 10^{-6}} = 44.1\,\text{At}$$

$$F_b = \Phi S_b = \Phi \cdot \frac{l_b}{\mu_0 \mu_r A_b}$$

$$= \frac{0.45 \times 10^{-4} \times 60 \times 10^{-3}}{4\pi \times 10^{-7} \times 1300 \times 90 \times 10^{-6}} = 18.4\,\text{At}$$

$$F_c = \Phi S_c = \Phi \cdot \frac{l_c}{\mu_0 \mu_r A_c}$$

$$= \frac{0.45 \times 10^{-4} \times 0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 150 \times 10^{-6}} = 119.3\,\text{At}$$

$$F = F_a + F_b + F_c = 44.1 + 18.4 + 119.3 = 181.8\,\text{At} = IN$$

$$I = \frac{181.8}{4000} = 45.4 \times 10^{-3}\,\text{A} = 45.4\,\text{mA}$$

Leakage Flux Leakage Coefficient

Leakage flux is the flux which follows a path not intended for it. In Fig.4.6 is shown an iron ring wound with a coil and having an air gap. The flux in the air-gap is known as the useful flux. Some of the flux leaks through air surrounding the iron ring this flux called leakage flux.

If, Φ_t = total flux produced ; Φ = useful flux available in the air-gap, then

$$\text{leakage coefficient } \lambda = \frac{\text{total flux}}{\text{useful flux}} \quad \text{or} \quad \lambda = \frac{\Phi_t}{\Phi}$$

The value of λ for modern electric machines varies between 1.1 and 1.25.

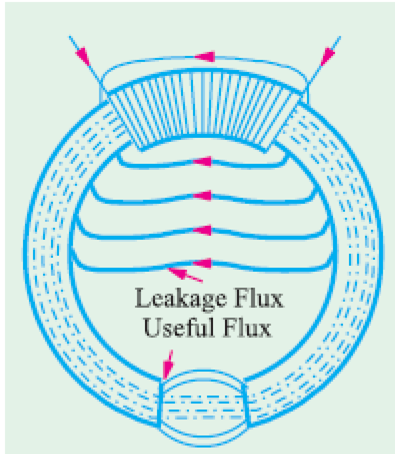


Fig.4.6